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# THIN-PLATE SPLINE QUADRATURE OF GEODETIC INTEGRALS

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ABSTRACT

Thin-plate spline functions - known for their flexibilty and fidelity in representing experimental data - are especially well-suited for the numerical integration of geodetic integrals in the area where the integration is most sensitive to the data, i.e. in the immediate vicinity of the evaluation point. Spline quadrature rules are derived for the contribution of a circular innermost zone to Stokes's formula, to the formulae of Vening Meinesz, and to the recursively evaluated operator $L_n$ in the analytical continuation solution of Molodensky's problem. These rules are exact for interpolating thin-plate splines.

In cases where the integration data are distributed irregularly, a system of linear equations needs to be solved for the quadrature coefficients. Formulae are given for the terms appearing in these equations. In case the data are regularly distributed, the coefficients may be determined once-and-for-all. Examples are given of some fixed-point rules. With such rules successive evaluation, within a circular disk, of the terms in Molodensky's series becomes relatively easy.

The spline quadrature technique presented here complements other techniques such as ring integration for intermediate integration zones.

Quadrature rules are sought approximating the contribution of a circular innermost zone to the evaluation of Stokes's formula, the formulae of Vening Meinesz, and the $L_1$ gradient operator in the series analytical continuation solution of Molodensky's problem. The rules are to be of the form

$$\zeta_I(x) = \frac{r_0}{\gamma_0} \sum_{i=1}^{N} a_i \Delta g(\alpha_i);$$

$$\begin{pmatrix} \xi_I(x) \\ \eta_I(x) \end{pmatrix} = \frac{1}{\gamma_0} \sum_{i=1}^{N} \begin{pmatrix} b_{1i} \\ b_{21} \end{pmatrix} \Delta g(\alpha_i);$$

$$L_{1,I} f(x) = \frac{1}{\gamma_0} \sum_{i=1}^{N} c_i \{f(\alpha_i) - f(x)\};$$

where $x = (x_1, x_2)$, $y = (y_1, y_2)$ are plane co-ordinates (with the 1-axis north and 2-axis east), $r_0$ is the radius of the innermost zone, and the subscript I indicates innermost zone contribution. $\{r_0\alpha_i\}$, $i=1,..,N$, are distinct points in the innermost zone (not all on a single straight line) where the data are given. The coefficients in each instance are chosen to make the integration exact for thin-plate splines with nodes at the $\alpha_i$, and exact for constant and linear functions in the nullspace of these splines. The thin-plate spline kernel function associated with function evaluation at $\alpha_i$ is $|y-\alpha_i|^2 \log_e |y-\alpha_i|$.

The quadrature weights for Stokes's formula are obtained from the solution of the linear equations

$$\sum_{i=1}^{N} a_i |\alpha_j - \alpha_i|^2 \log_e |\alpha_j - \alpha_i| + b_0 + b_1\alpha_{1j} + b_2\alpha_{2j} = d_j(x), \quad j = 1, \ldots, N;$$

$$\sum_{i=1}^{N} a_i = e_0(x); \quad \sum_{i=1}^{N} a_i\alpha_{1i} = e_1(x); \quad \sum_{i=1}^{N} a_i\alpha_{2i} = e_2(x);$$

where

$$d_j(x) = \frac{1}{2\pi}\iint_{|x-y|\leq 1} \frac{|y-\alpha_j|^2 \log_e |y-\alpha_j|}{|x-y|}\, dm(y) = -\frac{1}{9} + \frac{4}{9}|x-\alpha_j|^3;$$

$$e_0(x) = \frac{1}{2\pi}\iint_{|x-y|\leq 1} \frac{1}{|x-y|}\, dm(y) = 1;$$

$$e_1(x) = \frac{1}{2\pi}\iint_{|x-y|\leq 1} \frac{y_1 - x_1}{|x-y|}\, dm(y) = 0;$$

$$e_2(x) = \frac{1}{2\pi}\iint_{|x-y|\leq 1} \frac{y_2 - x_2}{|x-y|}\, dm(y) = 0.$$

Similar looking systems of equations give the quadrature weights for the spline approximation of Vening Meinesz'z formula and the $L_1$ operator. In the last case non-zero constant functions are not admissable, and the thin-plate spline kernel function needs some modification.

Numerical examples show that thin-plate spline quadrature can be very effective in evaluating the three integrals of Stokes, Vening Meinesz and the $L_1$ gradient operator.



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